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Tire Asymmetries and Pressure Variations in the Radt/Milliken Nondimensional Tire Model

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ABSTRACT

The Nondimensional Tire Model is based on the idea of data compression to load-independent curves. Through the use of appropriate transforms, tire data can be manipulated such that, when plotted in nondimensional coordinates, all data falls on a single curve. This leads to a highly efficient and mathematically consistent tire model.

In the past, data for slip angle and slip ratio has been averaged across positive and negative values for use with the transforms. In this paper, techniques to handle tire asymmetries in lateral and longitudinal force are presented. This is an important advance, since in passenger cars driving/braking data is almost always asymmetric and, depending on tire construction, lateral force data may follow likewise.

In addition, this paper is the first to explore the inclusion of inflation pressure as an operating variable in the Nondimensional Tire Theory. Inflation pressure affects the shape of the tire curves, notably the linear range stiffness and peak force friction coefficient. With this new variable, the operating conditions addressed by Nondimensional Tire Theory now include slip angle, slip ratio, inclination angle, normal load, surface friction coefficient and inflation pressure.

INTRODUCTION

There are many tire models in use around the world. They vary in complexity, scope and purpose. They can be based on detailed tire structural models, often employing finite element calculations, or they can be semi-empirical in nature. Purposes range from tire ride modeling, envelopment of road surface irregularities, tread shape analysis, rolling resistance calculations, tire dynamics, tire force & moment prediction and beyond. All tire models in use have their own strengths and weaknesses, and applications to which they are well-suited.

The Radt/Milliken Nondimensional Tire Model is a semi-empirical tire model used to predict net tire forces and moments. By semi-empirical, we mean that the model combines known operating conditions with some general assumptions of tire behavior to predict tire force and moment outputs [1]. Input variables are tire normal load, slip angle, inclination angle, slip ratio and surface friction coefficient. Outputs are lateral force, longitudinal force, aligning torque and overturning moment. The Nondimensional Tire Model follows SAE convention as defined in [2], in which Figure 1 appears.

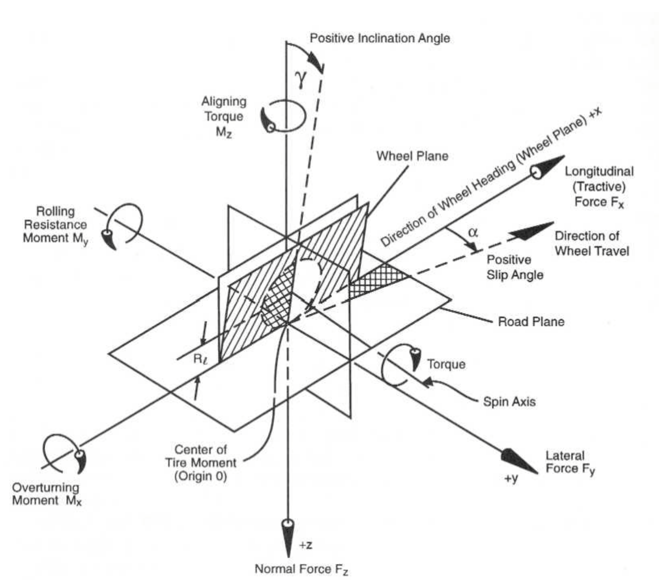

Figure 1. The tire coordinate system, definition of terms

The Nondimensional Tire Model is unique in that it transforms the raw tire data into dimensionless variables in such a way that compression of the data to load- and friction-independent curves results. Radt and Milliken first proposed the nondimensional model in 1960 [3]. They used Fiala's structural tire model [4] as the basis for development of the nondimensional transformations. In the simplest case, lateral force as a function of slip angle, the transformations are:

$$\bar{\alpha} = \frac{C \tan \alpha}{\mu F_z} \tag{1}$$

$$\bar{F} = \frac{F_y}{\mu F_z} \tag{2}$$

where α is the slip angle, F_z is the normal load, μ is the friction coefficient, C is the cornering stiffness and F_y is lateral force. $\bar{\alpha}$ and $\bar{F}$ are nondimensional variables.

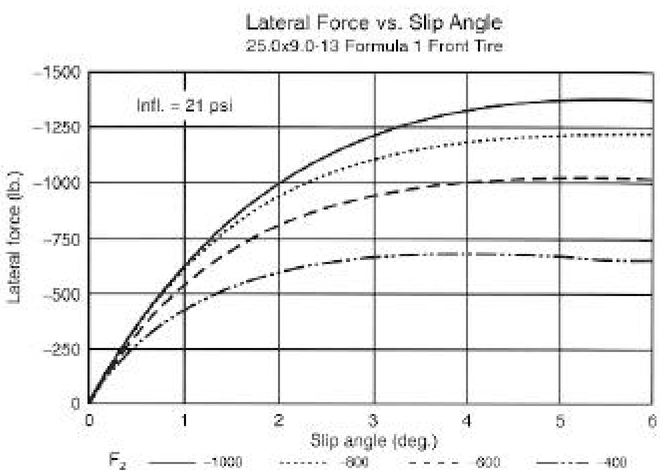

Figure 2. Typical Lateral Force vs. Slip Angle curves

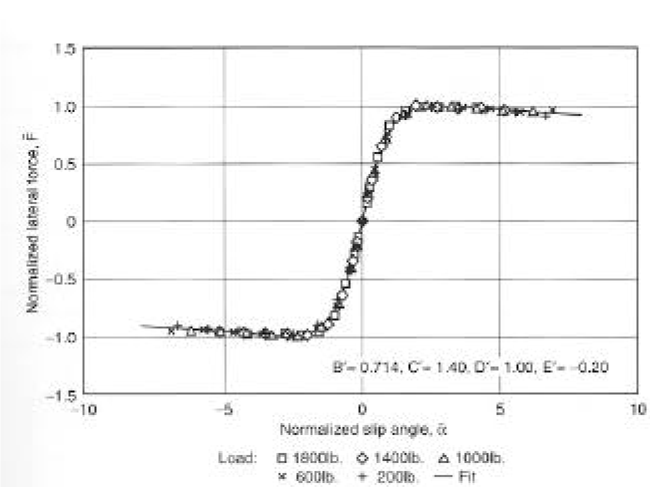

Figure 3. Nondimensional lateral force vs. slip angle

The result of these transformations is illustrated in Figures 2 and 3 from Milliken/Milliken [5].

The resulting load-independent curve in nondimensional coordinates (Figure 3) can then be fit by a curve such that $\bar{F} = f(\bar{\alpha})$. Early on, this was also based on the work of Fiala, such that $\bar{F}$ was a polynomial function of $\bar{\alpha}$. A significant improvement came in 1987 when Bakker and Pacejka [6] introduced the "Magic Formula". While Pacejka was using this "Magic Formula" to describe individual tire curves, Radt and Milliken applied it to the single curve in nondimensional coordinates [5] with much success. A more detailed look at the Magic Formula appears in the next section.

A good introductory summary to the Radt/Milliken Nondimensional Tire Model for a range of cases is given in Chapter 14 of Milliken/Milliken [5]. A more detailed summary is provided by Radt/Glemming [7]. Longitudinal force as a function of slip ratio is described, as are transformations for aligning torque, overturning moment, combined slip/camber and combined slip angle / slip ratio.

As yet the Radt/Milliken Nondimensional Tire Model is incomplete. Research on expanding it to general operating conditions is underway. This paper reports on two parts of this research, the modification of the nondimensional model to handle asymmetric tire data and the upcoming addition of pressure as an operating variable. These advances are presented after a review of the Magic Formula.

PROPERTIES AND APPLICATIONS OF THE "MAGIC FORMULA"

Before presenting the advances in nondimensional tire modeling, a more detailed review of the "Magic Formula" is worthwhile. Tire data curves are not well-described by polynomials, simple trigonometric or logarithmic functions. In the mid-1980s, Bakker, Pacejka and Nyborg [6] developed a formula which could be used to describe these tire data curves in functional form. It was given the name "Magic Formula" for its broad applicability to a wide range of tire curves.

In short, the Magic Formula is stated as:

$$f(x) = S_y + D\sin\left(C\tan^{-1}\begin{pmatrix} B(x - S_x)(1-E) \\ +E\tan^{-1}\left(B(x - S_x)\right) \end{pmatrix}\right) \tag{3}$$

where

? x is an input (such as slip angle)
? f(x) is a force or moment output (such as lateral force)
? S_x and S_y are shift factors which locate the center of the Magic Formula curve relative to the origin

? B, C, D and E are coefficients chosen to fit measured tire data

The coefficients B, C, D and E modify the shape of the curve. A few "rules of thumb" are now stated. The peak of the curve occurs with a value f(x) = D. The initial slope of the curve is given by the product BCD. B is related to the location of the peak along the x-axis. Coefficient E controls the shape of the curve in the vicinity of the peak. In practice, the effects of the coefficients are more complicated as they all have secondary effects. A much more detailed discussion of the parameters is given by Schuring [8], although this level of understanding suffices for the material that follows.

Radt and Milliken have applied this Magic Formula to Nondimensional tire curves [5]. In doing so, the coefficients B, C, D and E take on additional constraints. First, the shift coefficients S_x and S_y are set to zero. The product BCD is equal to one and the peak of the curve occurs at D = 1. These additional requirements are a result of the Nondimensional transformations.

Radt and Milliken [3,5,7] make several assumptions when preparing the data for fitting. Most importantly, they average tire forces or moments across positive and negative values of the independent variable to remove biases. In doing so, they assume the tire is symmetric with respect to the independent variable. The averaging and Nondimensional transformations are also assumed to eliminate the shift factors.

But what happens when the tires do not exhibit this symmetry with respect to the independent variable? Such asymmetries are common. In the case of pure longitudinal force as a function of the independent variable slip ratio, peaks on the driving side of the curve are often sharper than on the braking side of the curve. They may also occur at different friction levels. For racing tires used in oval track racing (left turns only) or unidirectional street tires, the tire may be constructed with one sidewall stiffer than the other, asymmetric tread patterns or asymmetric plies in the belts. These lead to differing tire performance in left and right turns.

In these cases, the averaging of data to eliminate asymmetries is unacceptable. New techniques to handle these asymmetries are now presented.

LATERAL FORCE = F(SLIP ANGLE): TRANSFORMATIONS

We now present methods to incorporate tire asymmetries into the Radt/Milliken Nondimensional Tire Model. We will use a simple case, lateral force as a function of slip angle (and whose transformations were presented in Equations 1 and 2), to explain the process. The process is representative of that used with more complex cases.

In this case, we allow the tire to have different shaped lateral force curves in the left and right turn. Typical raw data, collected on a tire testing machine at five different normal loads, is shown in Figure 4. We make no assumptions of symmetry about the origin.

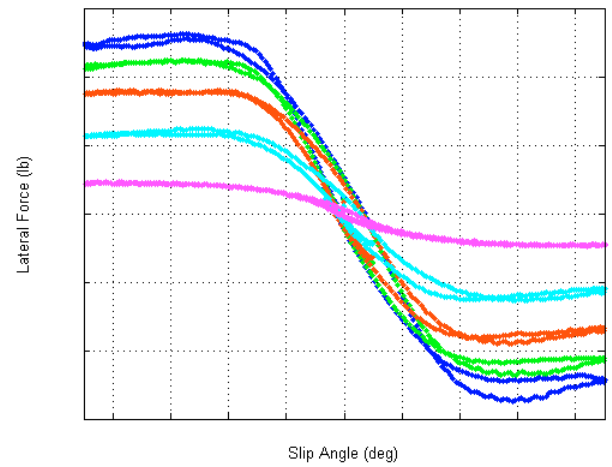

Figure 4. Raw lateral force versus slip angle data at several loads

The general approach taken to fitting asymmetries in this paper is to develop two separate sets of transformations, one for each side of the y-axis. By sharing certain properties, these transformations smoothly transition into a single, continuous curve.

The first step is to realize that Cornering Stiffness value used in Eq. 1 is defined as the slope of a lateral force versus slip angle curve at zero degrees slip angle. As such, it is common to both the left and right side transformations. A linear regression can be used to represent near-zero slip angle curves as straight lines for each load as shown in Figure 5. Each of the linear and zero degree regression terms can then be fitted with a polynomial to capture their variation with normal load.

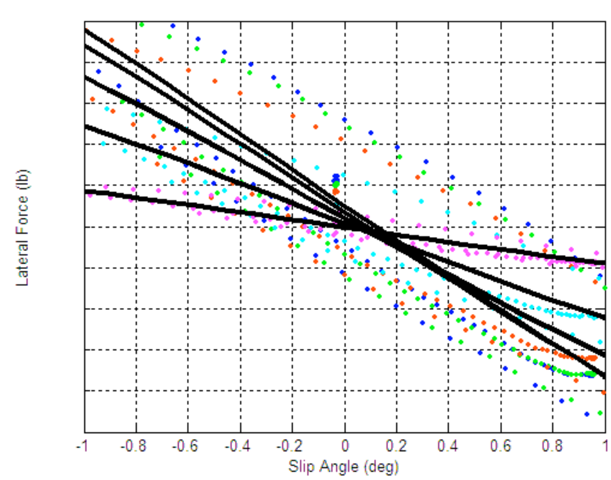

Figure 5. Determining Cornering Stiffness and Vertical Offset

The first degree terms describe the cornering stiffness and are used to calculate C in the Nondimensional Slip Angle transformation given by Equation 1.

The zero degree terms describe the vertical offset in the data at zero slip angle. These vertical shifts are subtracted from the raw data, leading to what we will term "zero-offset data". As a result, the zero-offset data curves pass through the origin. The zero-degree terms are common to both the left and right side transformations and replace the shifts S_x and S_y in the Magic Formula.

The next variable needed for the transformations of Equations 1 and 2 is the friction coefficient, μ. This would usually be obtained by finding the peak of a lateral force curve and dividing by the load. While this technique is still used, there are two variations. First, the zero-offset data is used instead of the original raw data. As a result, the "friction coefficient" is no longer the true peak, but rather the peak with the offset subtracted from the data.

Second, the friction coefficients are determined independently for the left and right sides of the graph as shown by the horizontal lines in Figure 6. This allows differences in behavior for left and right turns to be represented. It is a departure from the previous "averaging" technique.

As with cornering stiffness values, a polynomial is fit to the data to describe the variation in friction coefficient with normal load. The polynomial is used to select the appropriate value of μ in Equation 1.

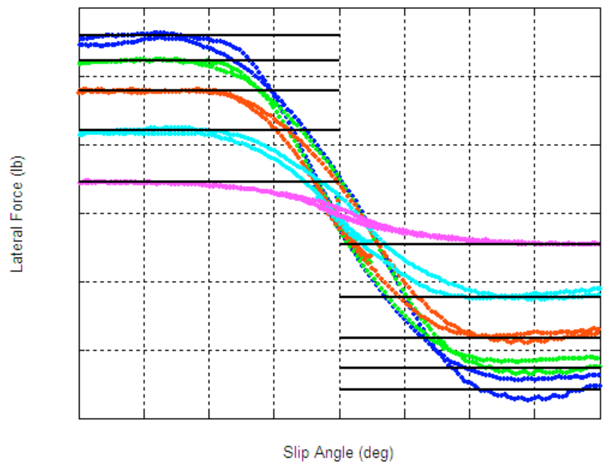

Figure 6. Peak values (friction coefficients) for zero-offset data.

LATERAL FORCE = F(SLIP ANGLE): MAGIC FORMULA

The result of applying the Nondimensional Tire Model transformations with identical cornering stiffness but different left/right friction coefficients to the zero-offset data is shown in Figure 7. The raw data collapses to a single, load-independent curve as expected.

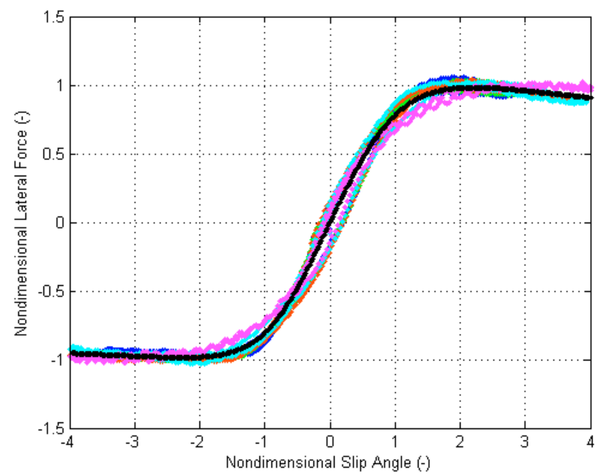

Figure 7. Tire data in Nondimensional Coordinates

In the past, a single instance of the Magic Formula has been used to describe this curve, but such a curve assumes symmetry about the origin. We cannot make this assumption. Instead, we can use two instances of the Magic Formula, one per side. In doing so, we specify two sets of coefficients: B_L, C_L, D_L and E_L for the left side of the graph and B_R, C_R, D_R and E_R for the right side.

Because we have used zero-offset data we are assured that the curves go through the origin. Thus, with S_x and S_y equal to zero we are assured of continuity between the left and right side Magic Formulas. Slope continuity, however, is not assured. Recall that the initial slope of the curve is given by the product BCD, which now is allowed to differ across the y-axis.

The potential for slope discontinuity is not particularly worrisome and can usually be ignored. This is because we already have the cornering stiffness common to both left and right side transformations to nondimensional slip angle. The resulting slope discontinuities are usually very small and can be deemed negligible.

In extreme cases, though, slope discontinuities can be non-negligible and have a noticeable effect when transmitted back through to expanded model data. This is undesirable. While the Nondimensional technique has not produced noticeable slope discontinuities in most cases, the likelihood increases with increasing tire asymmetry.

By allowing different values for B, C and D on the left and right sides we have allowed the product $B_L C_L D_L$ to differ from $B_R C_R D_R$, thus giving slope discontinuity at zero slip angle. There are several ways to eliminate or mitigate this slope discontinuity.

One option is to blend the two slopes over some range of ±slip angle. A possible blending function, defined over the range $-1 < \alpha < 1$, is:

$$BCD = \left(1 - \frac{\alpha + 1}{2}\right)(BCD)_L + \left(\frac{\alpha + 1}{2}\right)(BCD)_R \qquad (4)$$

This approach works well when the slope discontinuities are reasonably small.

A second option makes use of the additional restrictions placed on BCD by the nondimensional transformations. Since D is required to be equal to one (in practice, fits to the raw data produce D values which are very close to one, although not strictly equal to unity), we can assume the initial slope is given by the product BC. The product BC can then be blended across small slip angles instead of BCD.

A third option is to require the same value of C in both the left and right side Magic Formula expressions. This option works because of the secondary effects of coefficients B, C and D as expressed in [8]. When determining coefficients by regression, D_L and D_R will essentially unchanged as the peaks of the curve, while B_L and B_R will adjust themselves to deal with the common C coefficient.

In practice, the slope discontinuity is often so small that it is not apparent in the expanded data. If needed, however, these techniques can be used to reduce or eliminate the discontinuity.

SUMMARY OF THE NONDIMENSIONAL PROCEDURE WITH ASYMMETRIES

In summary, the following steps are used in calculating the Radt/Milliken Nondimensional Tire Model for lateral force as a function of slip angle:

1. Determine vertical offset at zero slip angle as a function of normal load
2. Create “zero-offset data” by subtracting vertical offset from raw data
3. Determine cornering stiffness as a function of normal load
4. Determine separate left and right side friction coefficients as functions of normal load
5. Apply transformations in Equations 1 and 2 to produce a load independent curve of nondimensional lateral force vs. nondimensional slip angle
6. Determine separate left and right side Magic Formula Coefficients
7. Decide whether or not to remove slope discontinuities at zero slip angle. If so, apply one of the techniques described above.

The result is that the entire set of lateral force curves are described by two sets of Magic Formula coefficients and polynomial functions of friction coefficient, cornering stiffness and vertical offset as functions of normal load. To expand the model, the above steps are applied in reverse order at a given load and slip angle.

The result of the model, expanded and overplotted with the raw data, is shown in Figure 8. As you can see, agreement between the raw data and the model is very good. This result is typical of the results produced by the Nondimensional technique.

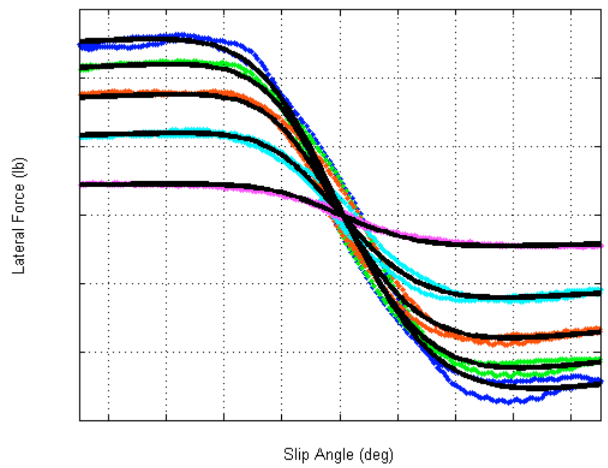

Figure 8. Raw data with Nondimensional model expansion

OTHER CASES

The same general approach can be applied to other cases, such as longitudinal force as a function of slip ratio and aligning torque as a function of slip angle. Asymmetries across the y-axis are handled by a similar series of steps, resulting in the fits shown in Figures 9 and 10. Once again, these are typical results.

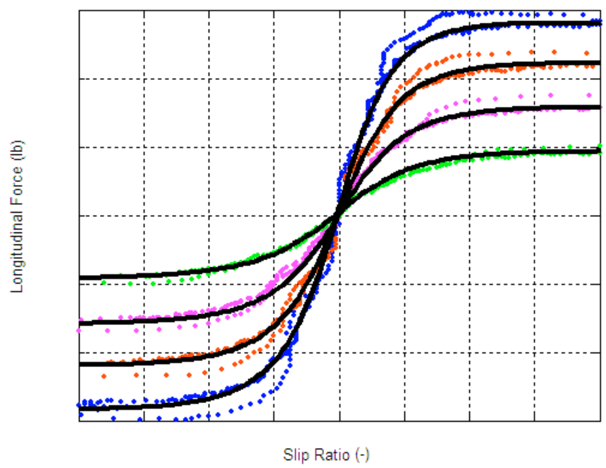

Figure 9. Longitudinal Force as a function of slip ratio

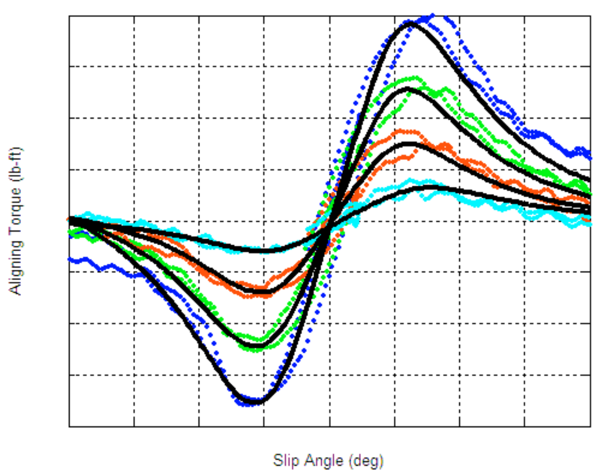

Figure 10. Aligning Torque as a function of slip angle

In the case of lateral force as a function of both slip angle and inclination angle (two variables), accounting for asymmetries becomes somewhat more complex. Milliken [5] presents the nondimensional transformations for combined slip and inclination angles as:

$$\bar{\beta} = \frac{\bar{\alpha}}{1 - \bar{\gamma}\,\mathrm{sgn}\,\alpha} \qquad (5)$$

$$FN = \frac{\bar{F} - \bar{\gamma}}{1 - \bar{\gamma}\,\mathrm{sgn}\,\alpha} \qquad (6)$$

In these expressions, the effect of inclination angle is accounted for by the transformations. Thus, vertical offset values are still measured at zero inclination angle only. The same goes for cornering stiffness. As before, since these values occur at zero slip angle and are used in both left and right side expressions. Camber stiffness, the slope of lateral force versus inclination angle at zero slip angle, likewise is used in both left and right side fits. Zero degree terms arising when calculating camber stiffness (essentially "camber offset") are not needed.

We note that in Equation 5, $\bar{\beta}$ combines slip and inclination angles into a single nondimensional parameter. One side of the curve describes "assisting inclination" in which the inclination angle increases total lateral force while the other describes "defeating inclination" in which the inclination angle works against lateral force from slip angle. This leads to inherent left/right asymmetries in the nondimensional plots. All plots of nondimensional lateral force versus $\bar{\beta}$ have asymmetries which cannot be ignored.

Friction coefficients for the zero-offset data are calculated as a function of both load and inclination angle via a response surface. The result of Equations 5 and 6 is a single, load-independent curve. Thus, the previous discussion of handling asymmetries beyond this point in the process remains unchanged.

Figure 11 presents some combined slip/inclination angle results at a non-zero inclination angle. It is clear in this graph how the inclination angle assists the negative lateral forces (left turn), but impair the positive lateral forces (right turn)

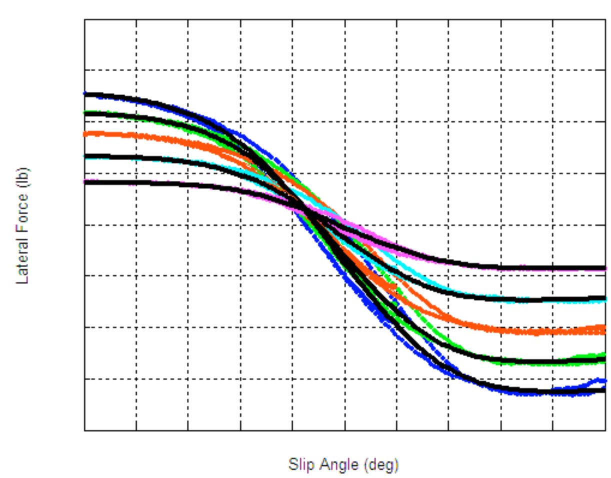

Figure 11. Combined slip/inclination angle modeling.

INFLATION PRESSURE MODELING

The Radt/Milliken Nondimensional Tire Model has been developed over the years to work with a specific set of operating variables. They are: normal load, slip angle, inclination angle, slip ratio and surface friction coefficient [7]. Of these, normal load and surface friction are always present. The nondimensional theory has been developed to account for non-zero pairs of the remaining three variables.

While research into Nondimensional Tire Theory continues with the goal of modeling all the above operating variables in non-zero conditions at the same time, there is a sixth variable of considerable interest. While not strictly an “operating variable”, inflation pressure is the easiest tire property to change and has a significant effect on tire performance. This is true in both the passenger car and racing worlds.

Figure 12 presents plots of lateral force vs. slip angle at 5 different loads and 4 different pressures. This raw data was gathered through measurement on a tire testing machine. To improve clarity, only one side of the plot (left turn) is shown. Horizontal lines are drawn to indicate the peak lateral force at a given combination of load and pressure. The peaks fall into five groups, one group per load, but vary by some amount as the inflation pressure changes.

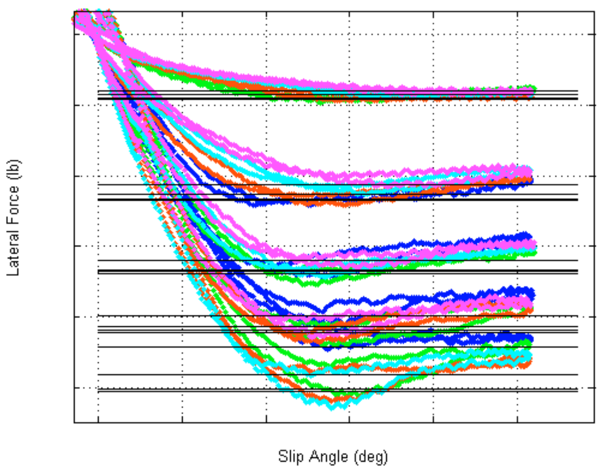

Figure 12. Pressure effect on lateral force.

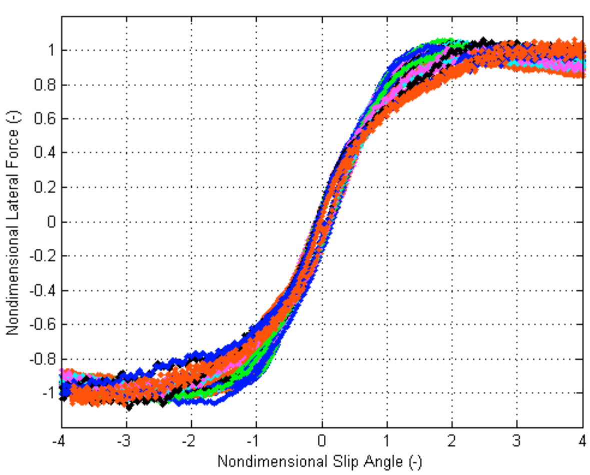

Figure 13. Nondimensionalized raw data

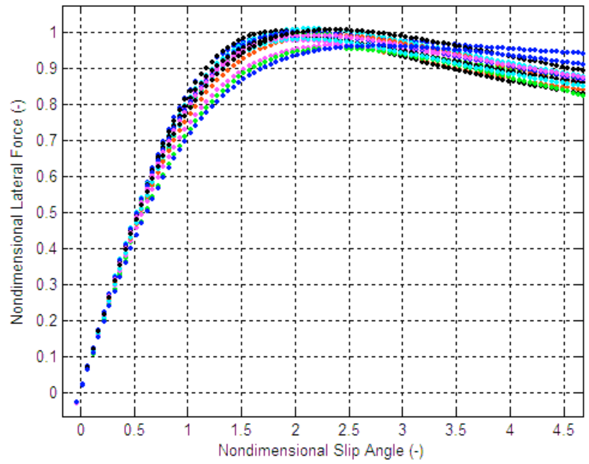

Figure 14. Magic Formula representation of nondimensional curves

The techniques described earlier can be used to transform this raw data into nondimensional form. The result is shown in Figures 13 (raw data) and 14 (Magic Formula representation of each load-pressure combination).

Figure 13 shows that the compression to nondimensional curves when inflation pressure is varied does not work as well as before. The compressed curve in Figure 13 is noticeably less compressed than the one presented in Figure 7. Figure 14 presents the same data smoothed by the Magic Formula. From this (again only one side shown for clarity) we see that the magnitude of the peak, location of the peak and shape of the curve around the peak vary as the pressure-load combination varies.

In itself, this is a useful result. The nondimensional curves can be studied to help determine variations in performance characteristics through the differences in the curves of Figure 14.

However, the results in Figure 13 and 14 are not consistent with Nondimensional Tire Theory’s premise of data compression. For curves at different pressures to collapse onto a single nondimensional curve, the nondimensional technique needs to be expanded.

At present, a preliminary technique has been developed which does a reasonable job of accounting for the inflation pressure effects. The approach is to take the Magic Formula descriptions in Figure 14 and first apply scaling factors to the height of the curve so that all peaks have exactly the same height. This can be done by dividing each curve by the Magic Formula D coefficient.

Then, a polynomial-based shift of the abscissa is performed such that the original Magic Formula curve is transformed into one which has a predetermined shape, such as shown in Figure 15. Here, the leftmost curve has been pre-selected with peak at a nondimensional slip angle value of 3. The rightmost curve is an existing curve.

By replacing the value x in Equation 3 with a low-degree polynomial function g(x) the rightmost curve can be shifted on top of the leftmost curve. This process is then repeated for all load-pressure combinations, resulting in a single compressed data curve. The abscissa transformations can then be cross-fit as a function of load and pressure such that intermediate values can be obtained during the expansion.

Research into including pressure variations into the Nondimensional Tire Model is in its infancy, although this first step shows that its inclusion is possible. A more complete description of techniques to handle pressure, either as an outgrowth of this concept or a completely new approach, will be presented in future papers as research progresses.

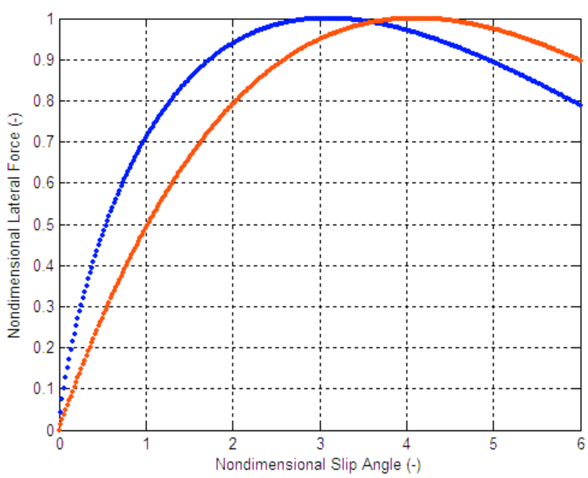

Figure 15. Scaling and transformation of Magic Formula curves

FUTURE WORK

There are several areas of active research on the Radt/Milliken Nondimensional Tire Theory. The search for an all-encompassing set of transformations which allow simultaneous non-zero slip angle, inclination angle and slip ratio is an ultimate goal. More comprehensive inclusion of inflation pressure as a model input is also being studied. This includes research on ways to capture pressure effects and the inclusion of pressure effects on outputs beyond lateral force (such as longitudinal force, aligning torque and combined operation).

Progress is also being made with respect to accounting for asymmetries in combined lateral-longitudinal force operation (i.e., the "friction ellipse"). Finally, the needs of the Nondimensional Tire Theory are not fully met by the "Magic Formula". A "More Magical Formula" (eventually with a better name) is being developed to overcome the deficiencies.

All this research is being conducted as part of the lead author's Ph.D. dissertation research. Additional papers are planned as results are achieved.

CONCLUSIONS

One of the major limitations of Nondimensional Tire Theory, namely the assumption of symmetric tire behavior, has been eliminated. Via the techniques presented in this paper, asymmetries in lateral force, longitudinal force, aligning torque, combined slip-inclination operation and other operating conditions can be handled. The technique is robust, compact and in the same spirit of the previously-existing nondimensional technique.

First steps toward the inclusion of inflation pressure as an input or operating variable have also been presented. While the technique works and gives smooth, mathematically consistent results, it also lays the groundwork for more elegant solutions as research continues. There is currently much effort into research on Nondimensional Tire Theory, and additional papers are planned.

ACKNOWLEDGEMENTS

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